Commodity Option Price Calculation Using CUDA Parallel Computing: Hydropower Risk Analysis Under Uncertainty In The Chili River Basin

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Summary
This article develops a model for the calculation of the price of options in commodities using a parallel computing platform based on the CUDA architecture, with the aim of significantly reducing the computational costs associated with the simulation and financial valuation processes. The proposed model is applied to the analysis of hydropower risk under conditions of uncertainty, taking the Chili River basin as a case study.
The methodology used integrates Monte Carlo simulation techniques and valuation of real options using trinomial trees, allowing estimating the prices of the main commodity variables linked to water resources, specifically electricity generation, human consumption and agricultural use. The approach considers the dynamic and stochastic behavior of these variables, as well as the influence of climatic factors generated from a hybrid stochastic model.
Deploying the model in a GPU environment demonstrates a significant acceleration in compute times versus traditional CPU deployments, especially as the complexity of the analysis and the number of nodes in the trinomial tree increases. The results show that the use of parallel computing CUDA is an efficient tool for risk analysis and optimal decision-making in the management of water resources under uncertainty.
Keywords
CUDA parallel computing; real options; commodities; hydropower risk; Monte Carlo simulation; trinomial tree; Chili River Basin

INTRODUCTION

The evolution of science and technology has been the result of the accumulated contributions of numerous researchers over time, whose work has been developed mainly from two complementary approaches: theoretical research and applied research. Both approaches have been fundamental for the advancement of scientific knowledge and for the resolution of specific problems of a social and technical nature, without excluding each other (Rubio Flores, 2010). In this context, the present work is mainly oriented towards applied research, without neglecting the theoretical foundations necessary to support the proposed methodological development.
In recent years, the development of computer systems has shown a marked trend towards the use of multiprocessor architectures, replacing traditional models based on a single processor. This evolution responds to the need to increase computational performance and reduce the costs associated with intensive data processing. Computational parallelism allows multiple calculations to be executed simultaneously, significantly increasing the processing power of computer systems (Rubio Flores, 2010).

In this scenario, graphics processing units have acquired a relevant role due to their inherently parallel architecture and their ability to execute a large number of operations concurrently. The CUDA technology developed by NVIDIA has made it possible to use GPUs in general-purpose applications, extending their use beyond the graphical field to scientific and technical problems that demand a high computational cost. This heterogeneous co-processing model, in which the CPU executes sequential tasks and the GPU accelerates intensive operations, has proven to be especially efficient in simulation and numerical calculation applications (Rubio Flores, 2010).

Risk analysis under uncertainty is a central element in decision-making associated with complex systems, such as those related to the management of water and energy resources. Such analysis requires the evaluation of multiple possible scenarios and the explicit consideration of the dynamic and stochastic behavior of the variables involved (Jorion, 2004). In this context, simulation techniques, particularly Monte Carlo simulation, have established themselves as fundamental tools for representing uncertainty and quantifying the risk associated with investment projects and production systems (Glasserman et al., 2000).

Likewise, the valuation of real options has emerged as an extension of the theory of financial options applied to real assets, allowing managerial flexibility and the possibility of making contingent decisions in the face of future evolution of risk variables to be incorporated into economic analysis (Dixit & Pindyck, 1994; López Lubián, 2000). This approach is especially suitable in natural resources and energy projects, where managers can choose between different alternatives for the use of the resource according to the conditions of the environment (Barría Quezada, 2008).

However, the application of methodologies based on Monte Carlo simulation and binomial or trinomial trees for the evaluation of real options implies the performance of a large number of calculations, which considerably increases the computational cost of the analysis (Hull & White, 1994). This limitation becomes more relevant as the complexity of the model, the number of nodes in the tree, and the number of simulated scenarios increase.

In this context, the use of GPU-based parallel computing platforms is presented as an efficient alternative to accelerate the simulation and evaluation processes of options in commodities. This article addresses the calculation of the price of options in commodities applied to the analysis of hydropower risk under uncertainty, using the Chili River basin as a case study. The proposed approach integrates the stochastic generation of climate scenarios (Podestá et al., 2005), energy commodity price models (Lucia & Schwartz, 2002; Escribano & Peña, 2002) and real options techniques implemented through trinomial trees, with the aim of improving computational performance and supporting optimal decision-making in the management of water resources for electricity generation, human consumption and agricultural use.

THEORETICAL FRAMEWORK

The theoretical framework of this research is based on concepts related to parallel computing, GPU-based processing architecture, risk analysis under uncertainty, stochastic simulation methods and the valuation of real options applied to energy commodities. These approaches support the proposed model for the efficient calculation of the price of options in contexts of high computational complexity.

Parallel computing architectures

The evolution of computer systems has been marked by the transition from sequential architectures to parallel processing models, motivated by the need to increase performance and reduce computational costs associated with large-scale problems (Rubio Flores, 2010). The classification of architectures proposed by Flynn allows us to understand this evolution based on the number of instruction and data flows that a system can process simultaneously.

The SIMD and MIMD architectures have become particularly relevant in scientific applications, as they allow the parallelism of data and tasks to be exploited. In particular, MIMD systems form the basis of modern high-performance environments, allowing the asynchronous execution of multiple sequences of instructions on different data sets, with shared, distributed, or hybrid memory schemes (Rubio Flores, 2010).

GPU-based parallel computing and CUDA architecture

Graphics processing units have evolved from devices dedicated exclusively to graphics processing to highly programmable platforms capable of executing general-purpose calculations. This paradigm, known as GPGPU, has made it possible to apply GPUs to scientific problems that require the massive evaluation of mathematical operations in parallel (Rubio Flores, 2010).

NVIDIA's introduction of the CUDA architecture cemented this approach by providing an accessible programming model based on high-level languages. In this model, the CPU acts as the control unit in charge of sequential tasks, while the GPU accelerates the sections with the highest computational load by simultaneously executing thousands of threads, organized in blocks and grids (Rubio Flores, 2010).

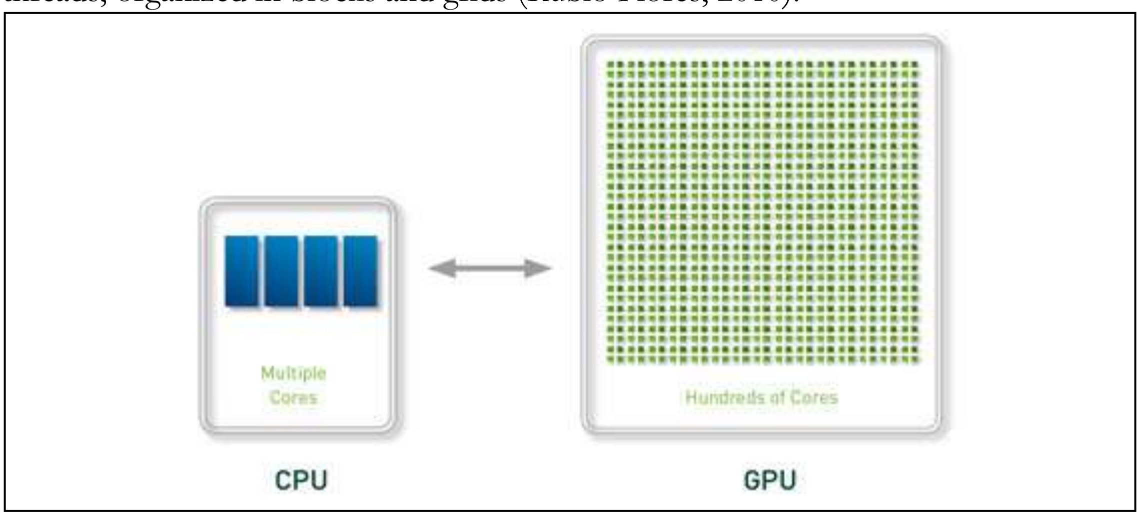

Figure 1. CPU–GPU interaction architecture in CUDA environment

This heterogeneous co-processing scheme is especially efficient for stochastic simulation and financial valuation applications, where a large number of independent scenarios need to be evaluated, such as in Monte Carlo simulation and in the construction of discrete trees for real options.

Risk analysis under uncertainty

Risk is defined as the possibility that a present action will generate multiple future outcomes, the occurrence of which cannot be determined with certainty. In most real problems, risk is subjective in nature, since the probabilities associated with events depend on incomplete information and the analyst's judgment (Jorion, 2004).

Risk quantification involves identifying the possible values of a random variable and the probability associated with each of them. In complex contexts, this quantification cannot be carried out by direct experimentation or simple analytical formulations, so probabilistic models and simulation techniques are used (Glasserman et al., 2000).

One of the most widely used metrics for measuring risk is Value at Risk, which estimates the maximum potential loss of an asset or project for a certain level of confidence and time horizon. This measure has established itself as a standard in financial and project risk management (Jorion, 2004).

Stochastic Simulation and Monte Carlo Simulation

Simulation is a fundamental tool for the analysis of systems under uncertainty, since it allows a model with different input values to be repeatedly evaluated, thus representing the set of possible scenarios. Among the most widespread methods is the Monte Carlo simulation, based on the random sampling of variables according to previously defined probability distributions (Glasserman et al., 2000).

This approach allows the complete distribution of the results of a model, facilitating the calculation of statistical indicators such as expected value, variance and extreme percentiles, which are essential for risk analysis and decision-making.

Real Options and Opportunity Assessment

Real options theory extends the principles of financial option valuation to the analysis of real assets, allowing for the explicit incorporation of managerial flexibility and the possibility of making contingent decisions in the face of future developments in risk variables (Dixit & Pindyck, 1994). This approach is particularly suitable in natural resources and energy projects, where investment decisions can be adapted to environmental conditions (López Lubián, 2000).

Real options can take a variety of forms, such as deferral, expansion, reduction, abandonment, or change of use of the resource. Its valuation allows us to capture the additional economic value generated by the possibility of modifying the investment strategy based on the behavior of the underlying variables (Barría Quezada, 2008).

Trinomial algorithm for option valuation

The trinomial algorithm is a widely used numerical method for the valuation of real options, as it allows the underlying variable to adopt three possible movements in each node of the tree: increase, permanence or decrease in value. This approach is particularly suitable for modeling Ornstein-Uhlenbeck-type mean-reverted stochastic processes (Hull & White, 1994).

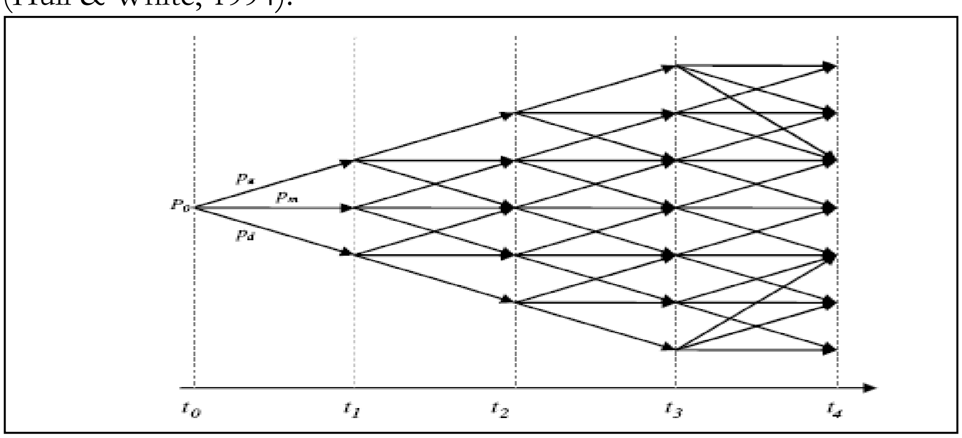

Figure 2. Solution Domain Structure of a Trinomial Tree

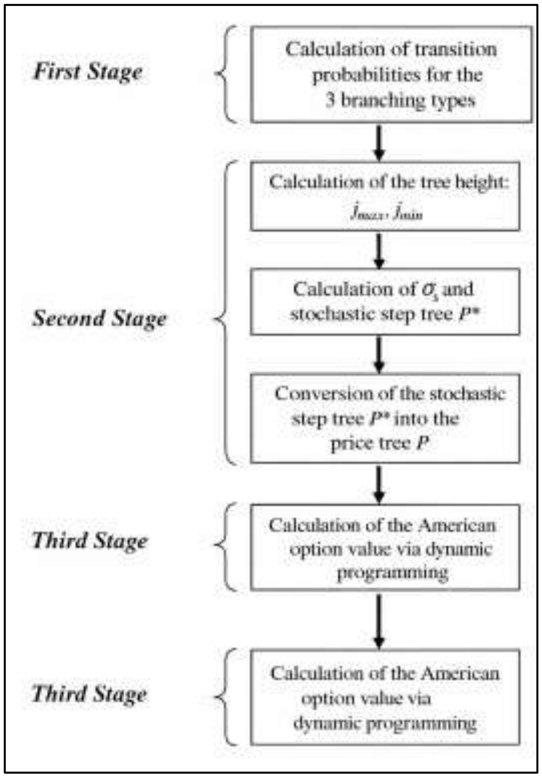

Figure 3. Trinomial algorithm flowchart

The construction of the trinomial tree requires defining transition probabilities that satisfy the conditions of expected value, variance, and normalization. Subsequently, through dynamic programming and a backward optimization process, the optimal investment strategy is determined, comparing the value of exercising the option against the value of waiting at each node of the tree (Hull & White, 1994).

Integrating the model into hydropower analysis

The application of the methodologies described above to the hydropower risk analysis allows the evaluation of different alternatives for the use of water resources, considering the generation of electricity, human consumption and agricultural use. The combination of stochastic simulation, real options and GPU-based parallel computing facilitates the efficient evaluation of a large number of scenarios, supporting optimal decision-making under conditions of uncertainty.

Methodology

The methodology used in this study is structured from a quantitative and experimental approach, aimed at evaluating the computational efficiency of the calculation of commodity option prices through the use of GPU-based parallel computing. The analysis integrates stochastic simulation techniques, real options assessment and parallel CUDA programming, applied to the study of hydropower risk under uncertainty in the Chili River basin.

Research Design

The research design is cross-sectional and experimental, since the variables involved in a single study period are analyzed through the controlled execution of computational simulations. The research is of an applicative and descriptive nature, since existing methodologies are implemented to evaluate their performance and the results obtained from their practical application are analyzed (Rubio Flores, 2010).

The dependent variable corresponds to the computational processing time required for the calculation of the price of options in commodities, while the independent variable is associated with the use of real options techniques implemented in a parallel computing environment CUDA.

Stochastic generation of climate scenarios

The first methodological component consists of the generation of synthetic climate scenarios, necessary to model the hydrological behavior of the Chili River basin. For this purpose, a hybrid stochastic generator is used, based on the methodology proposed by Rajagopalan and Lall, which combines a parametric approach to model the occurrence of precipitation with a non-parametric approach for the generation of the remaining climatic variables (Podestá et al., 2005).

This procedure allows the generation of multiple climatic trajectories consistent with the observed historical patterns, which serve as input for the simulation of hydropower variables and the subsequent evaluation of options.

Hydropower Commodity Price Models

Based on the climate scenarios generated, the price models of the main commodity variables associated with the use of water resources are estimated: electricity generation, human consumption and agricultural use. For this analysis, stochastic price models used in the literature for energy commodities are adopted, avoiding the formulation of new models and concentrating on the application of existing methodologies (Lucia & Schwartz, 2002; Escribano & Peña, 2002).

The behavior of prices is modeled considering stochastic processes with reversion to the mean, suitable to represent energy markets and natural resources subject to physical and regulatory constraints.

Monte Carlo Simulation

Monte Carlo simulation is used to represent the uncertainty associated with the input variables of the model and to generate a large number of possible scenarios. Each iteration of the simulation corresponds to a different realization of the stochastic variables, from which cash flows and relevant economic indicators are calculated (Glasserman et al., 2000).

This procedure allows the probability distribution of the results of the model to be obtained and constitutes the basis for risk analysis and the valuation of real options.

Calculating Risk Using Value at Risk

The risk associated with hydropower projects is quantified using the Value at Risk methodology. VaR is defined as the maximum potential loss that can occur at a given confidence level and over a specific time horizon (Jorion, 2004).

Formally, VaR is defined as the value that satisfies:

Equation (1)

$$P(V \leq VaR) = \alpha$$

Under the assumption of zero-mean yields, VaR can be expressed as:

Equation (2)

$$VaR = \alpha\sqrt{\sigma^2 \Delta t}$$

where it represents the level of significance, the variance of yields and the time horizon considered (Jorion, 2004).$\alpha \sigma^2 \Delta t$

When analyzing the value of the asset instead of the return, and considering that, the VaR of the value is expressed as:$V = N \cdot P$
Equation (3)

$$VaR_V = V_0 \cdot \alpha\sqrt{\sigma^2 \Delta t}$$

These expressions allow quantifying the risk associated with investment decisions and serve as a complement to the analysis of real options.

Valuation of real options using the trinomial algorithm

The valuation of real options is carried out by means of a trinomial algorithm, which allows the evolution of the underlying variable to be modelled by considering three possible movements in each node of the tree: increase, permanence or decrease in value (Hull & White, 1994).

The transition probabilities , and are defined in a way that satisfies the expected value, variance, and normalization conditions. For an unconstrained node, the probabilities are calculated based on:$p_u p_m p_d$
Equation (4)

$$p_u = \frac{1}{2}\left(\frac{\sigma^2 \Delta t}{\Delta x^2} + \frac{\kappa^2 x^2 \Delta t^2}{\Delta x^2} + \frac{\kappa x \Delta t}{\Delta x}\right)$$

Equation (5)

$$p_m = 1 - \frac{\sigma^2 \Delta t}{\Delta x^2} - \frac{\kappa^2 x^2 \Delta t^2}{\Delta x^2}$$

Equation (6)

$$p_d = \frac{1}{2}\left(\frac{\sigma^2 \Delta t}{\Delta x^2} + \frac{\kappa^2 x^2 \Delta t^2}{\Delta x^2} - \frac{\kappa x \Delta t}{\Delta x}\right)$$

The dynamics of the stochastic variable with mean reversion are described by:
Equation (7)

$$dP^* = -\kappa P^* dt + \sigma dW$$

The conversion to the real underlying variable is done by a time shift function, maintaining the tree structure and transition probabilities (Hull & White, 1994).

The option value is determined by dynamic programming, evaluating the value of exercising the option against the value of waiting at each node in the tree:
Equation (8)

$$V_{i,j} = \max\left(V_{i,j}^{ejercer}, V_{i,j}^{esperar}\right)$$

Implementation in CUDA parallel computing

The computational implementation of the model is carried out using the CUDA architecture, under a heterogeneous CPU-GPU coprocessing scheme. Sequential and control tasks are executed on the CPU, while the most computationally intensive operations, such as Monte Carlo simulation and trinomial tree node evaluation, are parallelized and executed on the GPU (Rubio Flores, 2010).

Using CUDA allows thousands of processing threads to be run simultaneously, significantly reducing the total calculation time as the number of simulated scenarios and the complexity of the model increases.

General outline of the methodological procedure

The complete methodological procedure integrates the generation of climate scenarios, stochastic price simulation, risk calculation using VaR and the valuation of real options implemented in a parallel computing environment.

This approach allows for the efficient evaluation of different alternatives for the use of water resources and supports optimal decision-making under conditions of uncertainty.

Data analysis

The data analysis in this research is aimed at evaluating the computational performance and applicability of real options valuation methodologies in the context of hydropower risk under uncertainty. To this end, the variables involved in the process of simulation and calculation of the price of options in commodities are considered, as well as the indicators associated with the efficiency of the implemented model.

Operational definition of variables

The dependent variable of the study corresponds to the computational processing time required for the calculation of the price of commodity options. This variable allows evaluating the impact of the use of GPU-based parallel computing compared to traditional CPU implementations, considering the complexity of the model and the number of nodes evaluated during the simulation.

The independent variable is represented by the application of real options methodologies for the valuation of hydropower commodities. This variable is operationalized through the calculation of the price of options associated with the generation of electricity, human consumption and agricultural use of water resources in the Chili River basin.

Analysis indicators

For the dependent variable, computational efficiency and the reutility of the implemented model are considered as main indicators. Efficiency is evaluated from the reduction of the calculation time obtained through the implementation in GPU, while reutility refers to the ability of the model to be applied to different scenarios and configurations without significant structural modifications.

In relation to the independent variable, the indicators considered include the price of the electricity generation option, the price of the human consumption option and the price of the agricultural use option. Additionally, the number of nodes served in the trinomial tree is analyzed, since this factor directly influences the computational complexity of the model.

Research hypothesis

The hypothesis raised in the study establishes that it is possible to significantly reduce the computational calculation time for the determination of the price of options in commodities through the use of a parallel development platform based on CUDA, applied to the analysis of hydropower risk under uncertainty.

This hypothesis is tested from the comparison of the execution times obtained in CPU and GPU environments, keeping the simulation and valuation methodologies used constant.

Level and type of research

The research level is experimental, since computational models are implemented and evaluated under controlled conditions, observing the effect of the use of parallel computing on the performance of the system. The type of research is applicative and descriptive, since existing methodologies are applied to analyze their behavior and the results obtained from their execution are described (Rubio Flores, 2010).

Research Design

The research design is transversal, since the analysis of the variables is carried out in a single study period through the execution of computational simulations. This approach allows for a timely assessment of the impact of the CUDA architecture on the processing time and efficiency of the calculation of real options.

Nature of the sample

The universe of the study is constituted by the set of hydrological scenarios possible in the time horizon considered. From this universe, the sample is defined by selecting the main prices of commodity options identified in the Chili River basin, corresponding to human consumption, agricultural demand and electricity generation.

The sampling method is aimed at experimenting the model on as many nodes of the trinomial tree as possible, in order to evaluate the behavior of the calculation time as the complexity of the analysis increases.

Analysis procedure

The data analysis procedure is structured in three main stages. First, synthetic climate scenarios are generated using the hybrid stochastic generator, which serve as the basis for the simulation of hydropower variables (Podestá et al., 2005). Second, the price models of the commodity variables are estimated and Monte Carlo simulations are run to obtain the distribution of cash flows and risk indicators (Glasserman et al., 2000). Finally, the trinomial algorithm is applied for the evaluation of real options and the computational processing times in the CPU and GPU environments are recorded.

The comparative analysis of the results allows us to evaluate the impact of the use of CUDA parallel computing on the efficiency of the calculation and to support the hypothesis raised.

RESULTS AND DISCUSSION

The results obtained from the implementation of the proposed model show that the use of GPU-based parallel computing through the CUDA architecture allows a significant reduction in the processing time required for the calculation of the price of options in commodities, compared to traditional implementations executed only on CPUs. This behavior is consistently observed as the complexity of the model increases, particularly when the number of simulated scenarios and the number of nodes evaluated in the trinomial tree increases.

The application of the Monte Carlo simulation, combined with the assessment of real options using trinomial trees, allowed to adequately represent the dynamic and stochastic behavior of the hydroenergy variables associated with the Chili River basin. In this context, options linked to electricity generation, human consumption and agricultural use showed a high sensitivity to uncertainty variables, especially those related to climate variability and energy commodity price dynamics.

From a computational point of view, the results confirm that the acceleration obtained by running on GPUs is more noticeable as the computational load increases. This behavior is explained by the ability of the GPU to simultaneously execute a large number of processing threads, which is especially efficient in the massive evaluation of independent scenarios, as occurs in the Monte Carlo simulation and in the calculation of node values within the trinomial algorithm (Rubio Flores, 2010).

However, the results also show that the total processing time includes the cost associated with transferring data between CPU memory and GPU memory. This overhead can partially attenuate performance gains in smaller-scale problems, although its relative impact decreases as the size of the problem and the number of options evaluated increases. This finding is consistent with what has been reported in the literature on parallel computing and GPGPU, where it is highlighted that the greatest advantages are obtained in applications with high computational intensity (Rubio Flores, 2010).

In terms of risk analysis, the integration of the Value at Risk methodology with the real options approach made it possible to complement the traditional risk assessment with an explicit assessment of flexibility in decision-making. While VaR provides a measure of the maximum potential loss for a given level of confidence, real options allow quantifying the value associated with the possibility of adapting investment decisions based on the future evolution of risk variables (Jorion, 2004; Dixit & Pindyck, 1994).

The results of the analysis confirm the complementarity between both approaches, as proposed in previous studies, where it is pointed out that the stochastic modeling used for the calculation of VaR can be consistent with that used in the evaluation of real options (Contreras & Fernández, 2003; Alessi, 2005). In the case of hydropower analysis, this complementarity is especially relevant, given that decisions to allocate water resources between electricity generation, human consumption and agricultural use are subject to physical, regulatory and climatic constraints.

From a water resource management perspective, the results obtained allow us to affirm that the proposed approach constitutes an adequate tool to support decision-making under uncertainty. The possibility of evaluating multiple scenarios and alternatives for the use of the resource, considering both the risk and the value of flexibility, offers a more complete view than traditional methods based solely on expected values.

Taken together, the results and their analysis confirm that the combination of stochastic simulation, real options valuation and parallel CUDA computing not only improves the computational efficiency of the calculation process, but also expands the analytical capabilities for hydropower risk management. These findings are consistent with previous research in the field of investment valuation under uncertainty and reinforce the relevance of the use of high-yield platforms for the analysis of complex natural resource systems (Barría Quezada, 2008; Andreu et al., 2009).

CONCLUSIONS

The present work developed a methodology for the calculation of the price of options in commodities through the use of parallel computing based on the CUDA architecture, applied to the analysis of hydropower risk under conditions of uncertainty in the Chili River basin. The proposed approach integrates stochastic simulation, evaluation of real options and parallel programming techniques, constituting a coherent methodological framework for the analysis of complex systems with high computational demand.

The results obtained allow us to conclude that the implementation of Monte Carlo simulation methodologies and evaluation of real options through trinomial trees in a GPU environment offers an efficient alternative to traditional implementations based exclusively on CPUs. The CUDA architecture facilitates the concurrent execution of a large number of operations, which is especially advantageous in problems characterized by massive scenario evaluation and high computational complexity.

Likewise, the study confirms the relevance of the real options approach for the management of water resources under uncertainty, by allowing for the explicit incorporation of flexibility in decision-making in the face of the future evolution of climatic, economic and energy variables. The application of the trinomial algorithm makes it possible to model stochastic processes with reversion to the mean and dynamically evaluate different alternatives for the use of water resources, such as electricity generation, human consumption and agricultural use.

The integration of the Value at Risk methodology with the analysis of real options made it possible to complement the risk assessment with an explicit assessment of the opportunities associated with operational flexibility. This complementarity offers a more complete view of the decision problem, overcoming traditional approaches based solely on risk measures or expected values.

From a methodological point of view, the work demonstrates that the combination of financial models, stochastic simulation and parallel computing is an adequate tool to support decision-making processes in complex hydropower systems. The proposed architecture is scalable and adaptable, which facilitates its application to other contexts of commodity analysis and natural resource management under uncertainty.

Finally, it is concluded that the main contribution of the research lies in the development and articulation of an efficient methodological scheme for the calculation of commodity option prices, rather than in obtaining specific numerical results. This approach lays the foundation for future applications and extensions aimed at empirical validation of the model and its implementation in real decision-making scenarios.

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